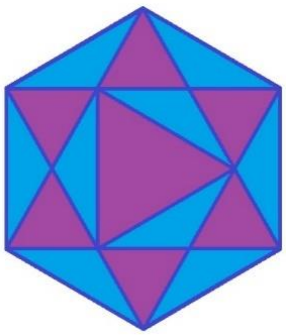
The Israeli Mathematics Olympiad for Grades 7 & 8
Final Round, 2021

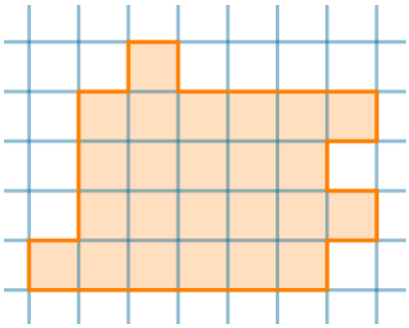
You are required to prove every statement and explain every answer (wherever it is relevant).

1. Miriam computed the squares of two consecutive natural numbers, added them and multiplied the result by 2. Prove that the resulting number is 1 more than a square number.



2. The hexagon in the drawing is regular. Which area is larger: the purple or the blue?

3. Divide this shape into 4 congruent parts:



4. In the following multiplication exercise, different letters represent different digits and identical letters represent identical digits. What was the original multiplication exercise?

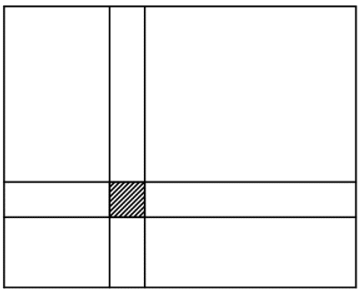
	×	א	ב	א	
			א	ב	
+		נ	ג	ל	
	ב	ל	ב		
	מ	ל	ש	ל	

5. Do there exist three distinct real numbers a, b, c, such that the three lines

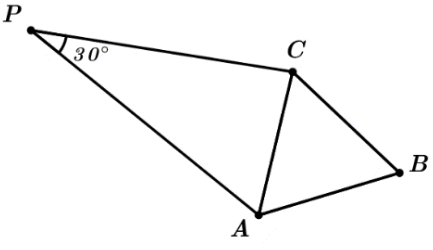
$$y = ax + a^2, \quad y = bx + b^2, \quad y = cx + c^2$$

all pass through one common point?

6. On a 43×47-cell square grid there is a blacked-out cell that doesn't touch the grid's perimeter. Taking the continuations of the sides of the cell until the points of intersection with the perimeter of the grid, we divide the shape into 8 smaller rectangles (as in the figure). Prove that one cannot construct a rectangle from the 8 smaller rectangles.



7. The triangle ABC is equilateral. We construct a triangle APC outwardly, such that $\angle APC = 30°$. Prove that one can construct a right-angled triangle from the segments AP, BP and CP.



Good luck!