סכומים

הוכיחו את הזהויות:

$$1. \sum_{i=1}^{n} id = \frac{n(n+1)d}{2}$$

$$2. \sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$$

$$3. \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$$

$$4.\quad 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$5. \sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

$$6. \sum_{i=1}^{n} i(i+1) \cdot \ldots \cdot (i+m) = \frac{1}{m+2} n(n+1) \cdot \ldots \cdot (n+m+1)$$

$$7. \sum_{i=1}^{n} \frac{1}{i(i+1) \cdot \ldots \cdot (i+m)} = \frac{1}{m}\left(\frac{1}{m!} - \frac{1}{(n+1) \cdot \ldots \cdot (n+m)}\right)$$

$$8. \prod_{i=1}^{n} (2^i - 1)!! = \frac{2 \cdot (2^n)!}{2^{2^n}}$$

$$9. \sum_{k=0}^{n} k \binom{n}{k} = n \cdot 2^{n-1}$$

$$10. \sum_{k=0}^{2n+1} (-1)^k \sqrt{\binom{2n+1}{k}} = 0$$

$$11. \sum_{k=0}^{2n+1} (-1)^k \cdot \frac{1}{\binom{2n+1}{k}^2} = 0$$

$$12. \sum_{k=0}^{n} (-1)^k \cdot \frac{1}{\binom{n}{k}} = (1 + (-1)^n) \cdot \frac{n+1}{n+2}$$

$$13^*. \sum_{k=0}^{n} \frac{1}{\binom{n}{k}} = \frac{n+1}{2^{n+1}} \sum_{k1}^{n+1} \frac{2^k}{k}$$